S. S. College, Jehanabad

B.Sc ~~Lect~~ (H) Physics Part II

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Lecture 2

Electromagnetic Theory

Cross product of two vectors

The cross product of two vector is defined as

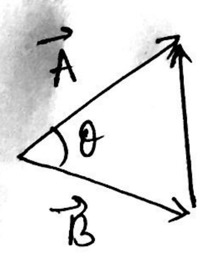
$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta \, \hat{n}$$

where $\hat{n}$ is a unit vector [vector of length 1] pointing perpendicular to the plane of $\vec{A}$ and $\vec{B}$.

* A hat (^) is used to designate unit vectors.

* There are two direction perpendicular to any plane "in" and "out".

* Exact direction perpendicular to the plane ~~is~~ can be determined using right-hand rule: Let your ~~fingers~~ fingers point in the direction of the first vector and curl around towards the second; then your thumb indicates the direction of $\hat{n}$.



* Here $\vec{A} \times \vec{B}$ points into the page.

* Here $\vec{B} \times \vec{A}$ points out of the page.

The $\vec{A} \times \vec{B}$ is itself a vector (hence the alternative name vector product).

* The cross product is distributive

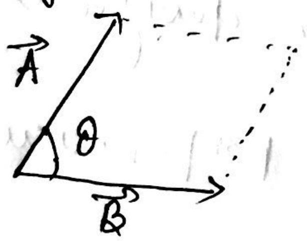
$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

* cross product is not commutative

$$\vec{B} \times \vec{A} \neq \vec{A} \times \vec{B}$$

But $\quad \vec{B} \times \vec{A} = -(\vec{A} \times \vec{B})$

Geometrically, $|\vec{A} \times \vec{B}|$ is the area of the parallelogram generated by $\vec{A}$ and $\vec{B}$ as shown below



If two vectors are parallel, their cross product is zero.

$$\therefore \quad \vec{A} \times \vec{A} = |\vec{A}|\,|\vec{A}| \sin\theta \, \hat{n}$$

$$\theta = 0^\circ \text{ and } 180^\circ$$

$$\vec{A} \times \vec{A} = 0$$

* To add vectors, add like components

Rule To multiply by a scalar, multiply each component.

$$a\vec{A} = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$$

Because $\hat{x}, \hat{y}$ and $\hat{z}$ are mutually perpendicular unit vectors.

$$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$$

$$\hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$$

Accordingly

$$\vec{A}\cdot\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Rule To calculate the dot product, multiply like components and add

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2$$

So, $$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

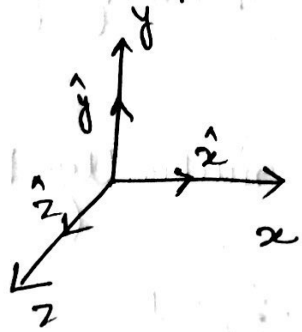
$$\left[\vec{A}\cdot\vec{A} = |\vec{A}|^2 \cos\theta = |A|^2 \cos 0^\circ = |A|^2\right]$$

* The dot product of $\vec{A}$ with any unit-vector is the component of $\vec{A}$ along that direction

eg. $\vec{A}\cdot\hat{x} = A_x$; $\vec{A}\cdot\hat{y} = A_y$; $\vec{A}\cdot\hat{z} = A_z$

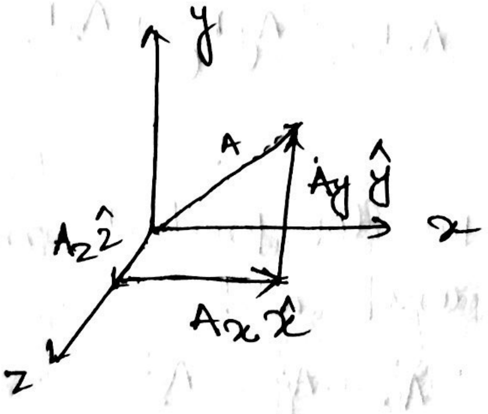
Vector Algebra: Component Form

Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be unit vectors parallel to the x, y and z axes respectively



An arbitrary vector $\vec{A}$ can be expanded in terms of these bases vectors

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$



The numbers A_x, A_y and A_z are called component of $\vec{A}$;. geometrically, they are the projection of $\vec{A}$ along the the three coordinate axes.

Addition of vector

$$\vec{A} + \vec{B} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) + (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$$
$$= (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

Similarly in cross product

$$\hat{x}\times\hat{x} = \hat{y}\times\hat{y} = \hat{z}\times\hat{z} = 0$$

$$\hat{x}\times\hat{y} = -\hat{y}\times\hat{x} = \hat{z}$$

$$\hat{y}\times\hat{z} = -\hat{z}\times\hat{y} = \hat{x}$$

$$\hat{z}\times\hat{x} = -\hat{x}\times\hat{z} = \hat{y}$$

Therefore

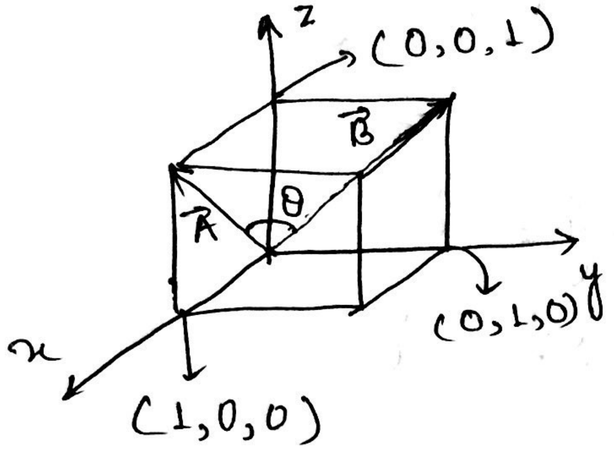
$$\vec{A}\times\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) \times (B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$$

$$= (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y}$$

$$+ (A_xB_y - A_yB_x)\hat{z}$$

The cross product can also be calculated ~~as~~ form the determinant whose first row is $\hat{x}, \hat{y}, \hat{z}$ whose second row is $\vec{A}$ (in component form) and whose third row is $\vec{B}$

* Find the angle between the face diagonals of a cube?



Let us assume a cube of side 1 and $\vec{A}$ and $\vec{B}$ are face diagonals.

$$\vec{A} = 1\hat{x} + 0\hat{y} + 1\hat{z}$$

$$\vec{B} = 0\hat{x} + 1\hat{y} + 1\hat{z}$$

So, in component form

$$\vec{A}\cdot\vec{B} = 1\cdot 0 + 0\cdot 1 + 1\cdot 1$$
$$= 1$$

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$|\vec{A}| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$$

$$|\vec{B}| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$$

$\Rightarrow \quad 1 = \sqrt{2}\sqrt{2}\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \cos^{-1}\frac{1}{2} \Rightarrow \theta = 60°$

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